Anticipated Attitude Motion of Skylab for a 1979 Revisit Mission

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The objective of the 1979 Skylab revisit mission is to place Skylab into a higher orbit or on a specified re-entry trajectory through the use of a teleoperator propulsion unit. This device will be attached via remote control to the Apollo docking port, provided Skylab's attitude motion is minimal. Digital computer simulations of current and predicted Skylab attitude motion have been developed. Gravity-gradient and aerodynamic torques are the primary disturbances acting on the vehicle. Limited observations of Skylab confirm results which indicate that it is presently oscillating about a gravity-gradient-stabilized mode. However, in October 1979, at least large oscillatory motion is predicted, but tumbling motion is highly likely. The expected higher solar activity and lower altitude of the spacecraft cause significant increases in atmospheric density and, therefore, larger aerodynamic torques. Uncertainties in future solar activity prevent accurate predictions of orbital life or the exact time of initiation of tumbling.

Introduction

THE Skylab spacecraft, launched in May 1973, is in a decaying orbit, which, if left alone, will probably re-enter the atmosphere by late 1980. Pieces of this large vehicle could land almost anywhere in the populated portion of the world due ot its orbital inclination of 50 deg. Therefore, late in October 1977, NASA decided to carry out a revisit mission in February 1980 on the fifth orbital test flight (OFT-5) of the Space Shuttle System. The mission was moved forward recently, in view of Skylab's orbital decay rate. It may take place as early as October 1979 on OFT-3. The purpose of this mission is to attach a teleoperator propulsion unit to the Skylab. This will change its orbit, either raise it or cause the spacecraft to re-enter over a specified remote ocean area. If the decision is to raise the orbit, this will allow the use of Skylab over a several year period as a base for experiments. The success of this mission depends on two uncertainties. First, the rate of orbital decay and magnitude of atmospheric torque are both functions of solar flare activity. Predictions of this activity are not altogether accurate. Therefore, there is a finite probability that Skylab will decay before October 1979. Of course, if this is the case, the mission will be cancelled. The second factor is that the mission plan is based on the assumption that Skylab will be in a gravity-gradient-stabilized mode. If this is not the case, it means that stabilizing the spacecraft may be required to insure mission success, or the mission would be cancelled.

The purpose of the work reported here is to develop a reasonable simulation which will permit anticipation of Skylab's motion for the revisit. Numerical work was done using an IBM 370/168 digital computer. Euler's moment equations for a rigid body were programmed along with the required transformation equations. Aerodynamic and gravity-gradient torques provided the inputs to the moment equations. A longitudinal density variation model about the Earth was used to determine more precisely the aerodynamic forces. Simulations indicate that Skylab is currently in such a gravity-gradient-stabilized orientation with bounded oscillatory motion away from nadir. There is also some angular motion about the longitudinal axis (nadir) at this time.

Simulations for late 1979 were run using a NASA solar activity model and predicted altitude profile. The orbital altitude for October 1979 is anticipated to be 345 km. Results indicate that Skylab will probably be exhibiting large oscillations or in a general state of tumble with minimum angular rates of 5.5 deg/min.

Dynamical Model

Three coordinate systems were used in the formulation of the problem – body, inertial, and orbital. The body coordinate system is attached to the spacecraft and rotates with it. The inertial coordinate system is nonrotating, while the orbital coordinate system rotates at the orbital rate ω_0. Therefore, the X_0 axis always points toward nadir (refer to Fig. 1 for nomenclature). Euler's equations were used to describe the motion of this rigid body.[1]

$$I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z = M_x$$
$$I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z = M_y$$
$$I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y = M_z \quad (1)$$

where ω_x, ω_y, ω_z are the body angular velocities about the x_b, y_b, and z_b axes, respectively. M_x, M_y, M_z are the moments acting about these axes. Principal moments of inertia for the Skylab configuration have been calculated by NASA to be $I_x = 3{,}694{,}680$ kg·m^2, $I_y = 3{,}767{,}828$ kg·m^2, $I_z = 793{,}321$ kg·m^2. Damping within Skylab was not modeled, since the associated time constant for this type of motion is thought to be several orbital periods, while the driving torques act continuously.

Since gravitational force acts in the orbital X_0 direction, a transformation is needed to express this force in the body frame. A typical Euler transformation is made between the body and the inertial coordinate systems, as illustrated in Fig. 2. The only difference between the orbital and inertial frames is a rotation about the Z_0 axis at the orbital rate ω_0. Define the angle γ as the orbital position angle which is equal to $\int\omega_0 dt$. Then define $\epsilon = \psi - \gamma$. The transformation matrix from

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the orbital to body coordinate system is then[1]

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} (C_\phi C_\epsilon - S_\phi C_\theta S_\epsilon) & (C_\phi S_\epsilon + S_\phi C_\theta C_\epsilon) & (S_\phi S_\theta) \\ (-S_\phi C_\epsilon - C_\phi C_\theta S_\epsilon) & (-S_\phi S_\epsilon + C_\phi C_\theta C_\epsilon) & (C_\phi S_\theta) \\ (S_\theta S_\epsilon) & (-S_\theta C_\epsilon) & (C_\theta) \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \tag{2}$$

where $C_\phi = \cos\phi$, $S_\psi = \sin\psi$, etc. The large matrix can be represented by α

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \alpha \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \tag{3}$$

In addition to a coordinate transformation, a transformation is needed between body rates and Euler rates. The Euler rates, $\dot\psi$, $\dot\theta$, $\dot\phi$ can be obtained by taking components of the body rates along the nonorthogonal axes

$$\dot\psi = (\omega_y C_\phi + \omega_x S_\phi)/S_\theta$$

$$\dot\theta = \omega_x C_\phi - \omega_y S_\phi$$

$$\dot\phi = [-C_\theta(\omega_y C_\phi + \omega_x S_\phi)/S_\theta] + \omega_z$$

Orbital rates referred to as the yaw rate $\dot\lambda$, pitch rate $\dot\zeta$, and roll rate $\dot\sigma$ are

$$\dot\lambda = \dot\phi S_\theta C_{(\pi/2-\psi+\gamma)} + \dot\theta C_{(\psi-\gamma)}$$

$$\dot\sigma = \dot\theta C_{(\pi/2-\psi+\gamma)} - \dot\phi S_\theta C_{(\psi-\gamma)}$$

$$\dot\zeta = \dot\psi + \dot\phi C_\theta - \omega_0 \tag{4}$$

If Skylab is in a gravity-gradient-stabilized mode, the Orbiter crew will see it with respect to the orbital coordinate system, as shown in Fig. 3. It is assumed that the Orbiter approaches Skylab from the rear. Orbital angles λ, σ, ζ, are obtained by integration of the associated rates, defined in Fig. 1. This orientation represents the initial state for the simulations presented below. Although the actual gravity-gradient orientation may have the solar arrays nearly coplanar with the orbital plane (due to the aerodynamic

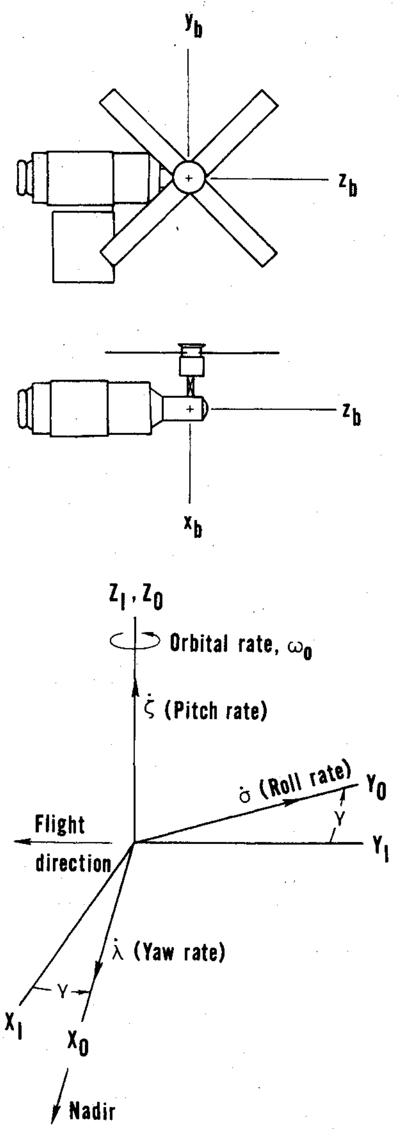

Fig. 1 Coordinate systems.

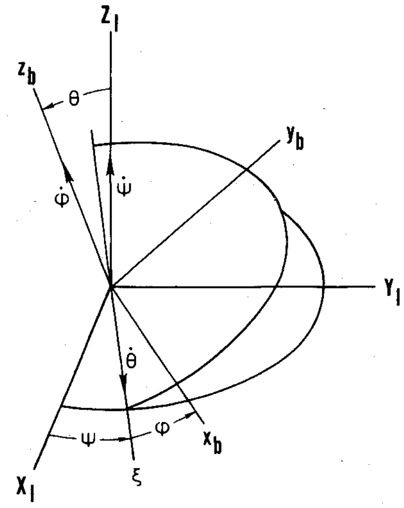

Fig. 2 Euler angles and rates.

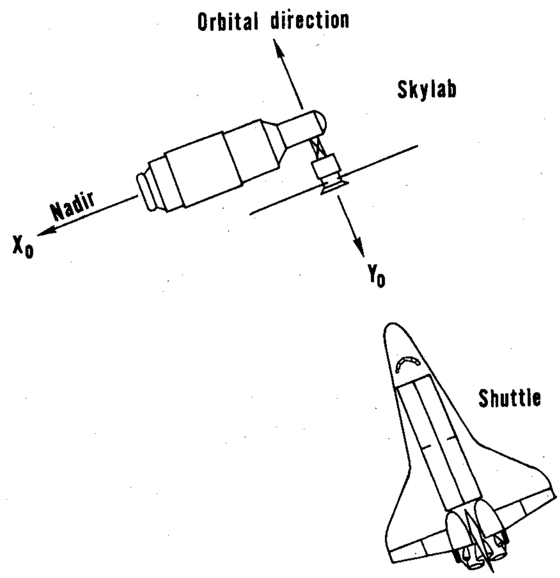

Fig. 3 Shuttle-Skylab relative positions in orbital plane.

forces), the orientation chosen here is more stable for beginning the simulations. A small spin rate about the body longitudinal axis is induced as the body searches for an aerodynamic equilibrium about the axis. Similar steady-state conditions, nevertheless, are achieveable.

Perturbing Torques

There are four potentially important environmental torques that act on a body in space. These are classified as aerodynamic, gravity-gradient, solar radiation, and magnetic. The inactive state and low orbit of Skylab leave only aerodynamic and gravity-gradient torques to be considered. Gravitational force acts in the direction of X_0, since this axis always points toward nadir. The basic principle of gravity-gradient stabilization is that a body in a gravitational field, having one moment of inertia less than the other two, will experience a torque tending to align the axis of least inertia with the field direction.[1] If $\boldsymbol{R_0}$ is the radius vector from the center of the Earth to the center of the orbital coordinate system, its components are easily determined in the body coordinate system using Eq. (3)

$$\begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} = \alpha \begin{bmatrix} -R_0 \\ 0 \\ 0 \end{bmatrix}$$

or

$$R_x = -R_0(C_\phi C_\epsilon - S_\phi C_\theta S_\epsilon)$$
$$R_y = -R_0(-S_\phi C_\epsilon - C_\phi C_\theta S_\epsilon)$$
$$R_z = -R_0 S_\theta S_\epsilon \tag{5}$$

Moments due to gravity-gradient torques about the body axes are obtained as

$$L_x = (3\mu/R_0^5) R_z R_y (I_z - I_y)$$
$$L_y = (3\mu/R_0^5) R_x R_z (I_x - I_z)$$
$$L_z = (3\mu/R_0^5) R_x R_y (I_y - I_x) \tag{6}$$

where μ is the gravitational constant of the Earth. These represent part of the inputs to Euler's equations. Moments due to the aerodynamic force must also be determined.

The orbital velocity V_0 is in the minus Y_0 direction. Hence, components of velocity about the body axes are determined using Eq. (3)

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \alpha \begin{bmatrix} 0 \\ -V_0 \\ 0 \end{bmatrix}$$

or upon expansion

$$V_x = -V_0(C_\phi S_\epsilon + S_\phi C_\theta C_\epsilon)$$
$$V_y = -V_0(-S_\phi S_\epsilon + C_\phi C_\theta C_\epsilon)$$
$$V_z = +V_0 S_\theta C_\epsilon \tag{7}$$

Moments due to the aerodynamic forces about the body axes are determined from

$$A_x = c_x q A_{\text{ref}} D_{\text{ref}}$$
$$A_y = c_y q A_{\text{ref}} D_{\text{ref}}$$
$$A_z = c_z q A_{\text{ref}} D_{\text{ref}}$$

where $q = (\frac{1}{2})\rho V_0^2$, A_{ref} is the reference area, and D_{ref} is the reference diameter. Determination of the atmospheric density ρ will be discussed later. The aerodynamic drag moment coefficients c_x, c_y, c_z were obtained from a NASA aerodynamic drag model.[2] A Fourier series curve-fit formula for c_x, c_y, c_z was derived as a function of the angle of attack α_a and angle ϕ_a about the longitudinal axis of Skylab. These angles are computed as

$$\alpha_a = \cos^{-1}(V_z/V_0) \qquad (0 \le \alpha_a \le 180 \text{ deg})$$

$$\phi_a = \tan^{-1}(V_y/-V_x) \qquad (0 \le \phi_a \le 360 \text{ deg})$$

The complete set of the moment components was used as input to Euler's equations. Thus, the right side of Eq. (1) consists of gravity-gradient and aerodynamic moments

$$M_x = L_x + A_x$$
$$M_y = L_y + A_y$$
$$M_z = L_z + A_z \tag{8}$$

Atmospheric density is the most influential parameter with respect to the attitude of Skylab. This was calculated using the Jacchia (1970) model about the Earth at various orbital altitudes at the equator.[3,4] Although Skylab is in a 50-deg inclined orbit, which means it will pass over latitudes of up to 50 deg, equatorial densities were used to approximate density variations around the Earth. Test runs were made with a more accurate atmospheric model which took into account the latitude variation. Results of these runs show no significant difference with respect to those run with the simplified density model. Thus, density variation could be simply modeled as a function of longitude, i.e., a sinusoidal variation. This sinusoidal distribution is due to the density difference between the sun side of the Earth and the shaded side at a given altitude.

Simulation and Results

Simulations were done using an IBM 370/168 computer. The program uses a Hamming predictor-corrector method for integration. Due to the required accuracy, many of the runs used an excessive amount of computation time. Parameters varied when doing the simulations were orbital altitude and date. Density is a decreasing function of the orbital altitude, the form of which is indicated in Fig. 4. This is the anticipated curve for October 1979. Density is time-dependent because of

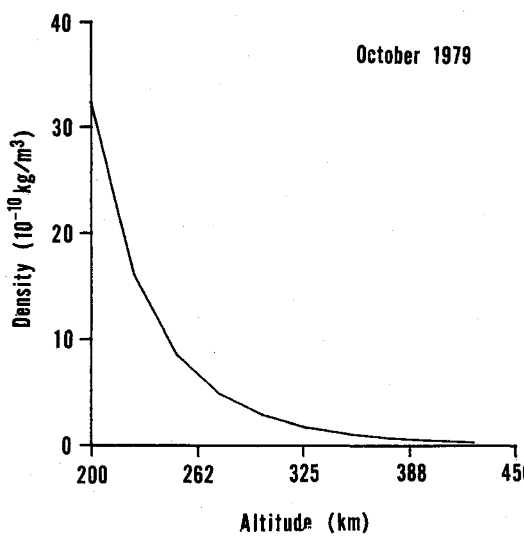

Fig. 4 Average density as a function of altitude.

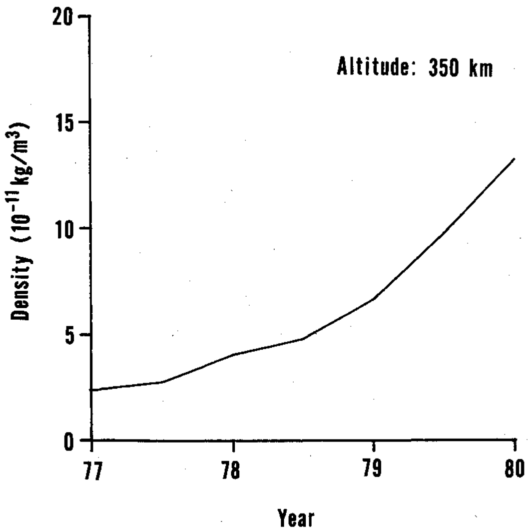

Fig. 5 Average density dependence on solar activity cycle.

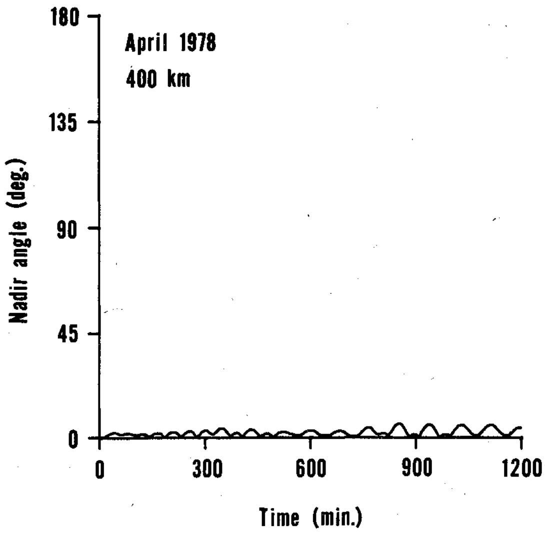

Fig. 7 Attitude motion for mid-1977 period.

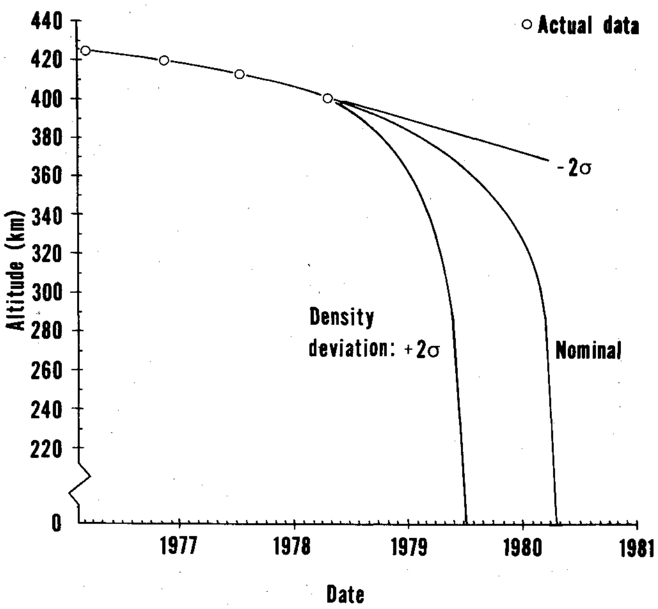

Fig. 6 Prediction of Skylab orbital decay.

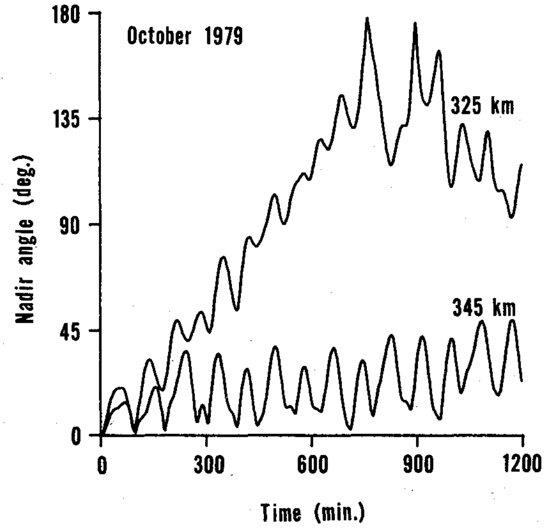

Fig. 8 Predicted motion of Skylab at mission time.

the 11-year cycle of solar activity among other cycles. Figure 5 offers a density profile for the time period of interest at an altitude of 350 km. This is the primary influencing factor on decay and attitude motion. The peak of solar activity is predicted to occur near the mission date.[5] Simulations indicate this will cause an increase in aerodynamic torques probably sufficient to tumble Skylab. Uncertainty in solar activity predictions is significant and does require a consideration of the effects of deviations from the nominal case.

NASA has provided a curve of anticipated orbital decay, an updated version of which is given in Fig. 6 with the associated 2σ deviations. The nominal curve predicts an orbital altitude of about 345 km at mission time. If the decay rate were to exceed nominal, the lower altitude will have a significant effect on the attitude motion. On the other hand, a lower decay rate will mean a higher altitude and more stable Skylab at mission time. Observations over the next few months may give important insight into the actual trend of decay.

Simulations were carried out for different points in time between May 1977 and February 1980 using the predicted density model just mentioned. Figure 7 shows the results for April 1978. The nadir angle is measured between the positive z_b axis and nadir. Thus, in April 1978, Skylab experienced an oscillation of less than 7 deg from the gravity-gradient orientation. Furthermore, simulations also indicate that the angular rate and amplitude about the longitudinal z_b axis are very small. A simulation for an altitude of 345 km using the predicted solar activity data for October 1979 resulted in the lower curve of Fig. 8. The vehicle was initially in a gravity-gradient orientation. This curve indicates that oscillatory motion with a 45 deg half-angle will be persistent. Angular rates associated with this motion are up to 5.4 deg/min. The upper curve of Fig. 8 shows the expected motion if Skylab's altitude was 20 km ($\approx$10 N-mi) lower at mission time. This could easily occur if the solar activity were to increase. The resulting motion is a slow tumble. The significance of this curve is that tumbling motion will occur even if the spacecraft is initially in a gravity-gradient orientation. In the actual case, of course, Skylab will have gone through a long period of reorientation and oscillation before it reaches the mission date. In fact, there may be a period in which Skylab reaches a new equilibrium orientation with respect to a combination of gravity-gradient and aerodynamic torques. It may even be relatively stable in this orientation. Such a state has been investigated with no success. Different initial orientations were tried using the perturbations associated with the atmospheric model near the mission date. Similar results were obtained, i.e., large oscillatory motion or a state of tumble was reached.

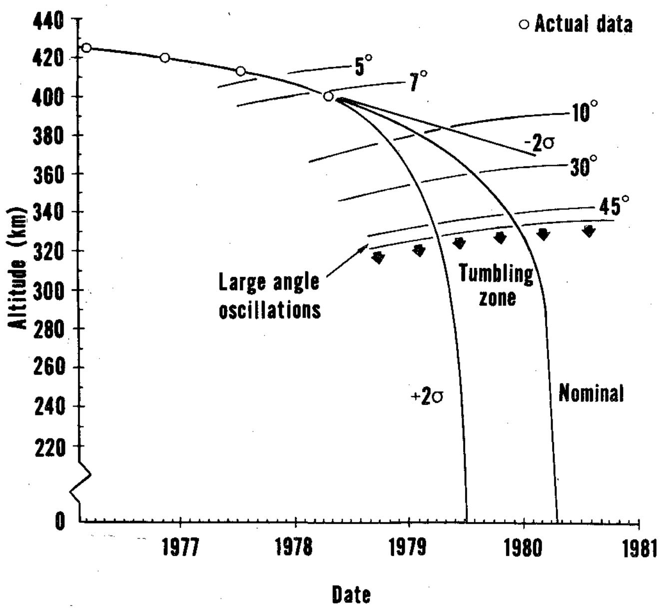

Fig. 9 Summary of attitude motion simulations.

Simulations run for dates between early 1977 and February 1980 indicate that the amplitude of oscillations away from the nadir direction build up gradually. Figure 9 summarizes the anticipated motion for this period. The nominal trajectory and $\pm 2\sigma$ deviations are shown. The somewhat horizontal lines indicate the expected degree of oscillatory motion about the nadir, which increases as orbital altitude decreases. Rates associated with the oscillatory motion are given in Table 1. The implication of this is clear. Skylab will slowly leave a gravity-gradient orientation state and eventually enter a state of general tumble. The crossover point in time between simple oscillations away from the gravity-gradient mode and tumble is not clear-cut because of the uncertainties in solar activity and orbital decay.

Conclusions

Attitude motion of Skylab under the influence of gravity-gradient and aerodynamic torques has been simulated for the time period of May 1977 to February 1980. Results indicate that it is now in a stable gravity-gradient mode with its longitudinal axis, z_b pointed toward Earth. It will remain in this orientation with only small oscillations until early 1979. After that time these oscillations will grow in amplitude. Eventually, Skylab will tumble and later re-enter the atmosphere. The timing of these events is uncertain, but a revisit mission should anticipate dealing with a tumbling vehicle.

The most sensitive parameter is orbital altitude. A variation of 20 km from that expected could determine the success or failure of the mission. A delay beyond October 1979 might have an associated increase in attitude motion, while an earlier mission would enhance the chances of success.

In conclusion, if NASA goes forward with the mission, some oscillatory motion is almost certain and must be anticipated. Associated operational procedures and hardware should be anticipated for this situation. Since uncertainties in altitude and attitude motion will persist to within a few months of this mission, docking with a tumbling Skylab or detumbling it should be assumed as a requirement for mission success.

Table 1 Rates associated with Skylab oscillatory motion

Oscillation, deg	Transverse nadir rate, deg/min	Longitudinal roll rate, deg/min
5	0.2	2.3
7	0.3	2.4
10	0.4	4.2
30	0.6	4.4
45	1.8	5.4
tumbling	2.4^+	5.5^+

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